One-loop renormalisation for the second moment of GPDs with Wilson fermions

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Overview

- Introduction
- Second moment in lattice perturbation theory
- Operators and mixing
- Examples for renormalisation factors
- Summary & outlook

Introduction

- Generalised parton distributions (GPDs) are very interesting
  They contain more information about the hadron structure than the usual structure functions

- GPDs unify parametrisations for large class of hadronic correlators, e.g. form factors and distribution functions

  They contain informations about the transverse structure of hadrons and the orbital angular momentum carried by quarks and gluons

- GPDs well-defined QCD objects systematically studied in perturbation theory (e.g. Geyer, Müller, Robaschik,..., Ji, Radyushkin,...)

- Experimental access: $ep \longrightarrow ep\gamma$, $ep \longrightarrow ep\pi^+\pi^-$
  first data from e.g. HERMES show some evidence

- Need complementary information from lattice QCD

Able to calculate non-forward matrix elements of local composite operators

($\Delta = p - p'$, $\overline{p} = \frac{p+p'}{2}$, $(\cdots)$: index symmetrisation and subtraction of trace terms)

$$\langle p'|\mathcal{O}^{\mu_1\cdots\mu_n}|p\rangle = \bar{\psi}(p')\gamma^{(\mu_1}\psi(p)\sum_{i=0}^{\left[\frac{n-1}{2}\right]} A_{n,2i}(\Delta^2)\Delta^{\mu_2}\ldots\Delta^{\mu_{2i+1}}\overline{p}^{\mu_{2i+2}}\ldots\overline{p}^{\mu_n)}$$

$$-\frac{1}{2M}\bar{\psi}(p')\,i\sigma^{\alpha(\mu_1}\Delta_\alpha\psi(p)\sum_{i=0}^{\left[\frac{n-1}{2}\right]} B_{n,2i}(\Delta^2)\Delta^{\mu_2}\ldots\Delta^{\mu_{2i+1}}\overline{p}^{\mu_{2i+2}}\ldots\overline{p}^{\mu_n)}$$

$$+C_n(\Delta^2)\mathrm{Mod}(n+1,2)\frac{1}{M}\bar{\psi}(p')\psi(p)\Delta^{(\mu_1}\ldots\Delta^{\mu_n)}$$

Leading twist 2 operators: $\mathcal{O}^{\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1}\bar{\psi}\gamma^{(\mu_1}\overleftrightarrow{D}^{\mu_2}\ldots\overleftrightarrow{D}^{\mu_n)}\psi$

Relate generalised form factors A, B, C to moments of GPDs

$$\int_{-1}^{1} dx\, x^{n-1} H_q(x,\xi,\Delta^2) = \sum_{i=0}^{\left[\frac{n-1}{2}\right]} A_{n,2i}(\Delta^2)(-2\xi)^{2i} + \mathrm{Mod}(n+1,2)C_n(\Delta^2)(-2\xi)^{2n}$$

$\xi = -n\cdot\Delta$, $n\cdot\overline{p} = 1$

Example shown with: $H(x,0,0) = q(x)$

First numerical lattice results 03: QCDSF and LHPC

Latt04: Zanotti, Pleiter,...

- Important: relate lattice results to continuum:

  $\longrightarrow$ need renormalisation factors

- Non-perturbative determination of Z-factors finally preferable

  But

  - Computationally rather complicated (concerning clear signals)
  - Some aspects (explicit dependence on the lattice spacing a, mixing, ...) can be studied in lattice perturbation theory rather naturally

- Complications

  H(4) less stringent than O(4): possibilities for mixing increase

  Mixing with operators containing total derivatives starts from $n = 3$ (2nd moment)

- This talk: First results for the Z-factors of the 2nd moment of GPDs (Wilson fermions in Feynman gauge)

Second moment in lattice perturbation theory

We consider matrix elements of the following operators

$$\mathcal{O}^{DD}_{\mu\nu\omega} = \overline{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\omega \psi \quad (1)$$

$$\mathcal{O}^{\partial\partial}_{\mu\nu\omega} = \partial_\nu \partial_\omega \left(\overline{\psi}\gamma_\mu \psi\right) \quad (2)$$

$$\mathcal{O}^{\partial D}_{\mu\nu\omega} = \partial_\nu \left(\overline{\psi}\gamma_\mu \overleftrightarrow{D}_\omega \psi\right) \quad (3)$$

$$\overline{\mathcal{O}}_{\mu\nu\omega} = \partial_\nu \left(\overline{\psi}\left[\gamma_\mu, \gamma_\omega\right]\psi\right) \quad (4)$$

In addition operators with $\gamma_\mu \to \gamma_\mu \gamma_5$, e.g. $\mathcal{O}^{\partial D,5}_{\mu\nu\omega}$

Mixing problem for form factors studied by Shifman and Vysotsky (1981)

They derived mixing matrices for anomalous dimensions only between operators (1) $\leftrightarrow$ (2)

Operators (3,4) are special for GPD and the transformation properties under hypercubic group H(4)

Computation

We have expanded our Mathematica package developed for forward matrix elements (Wilson, clover, overlap)

- Calculation performed in Kawai scheme (dimensional regularisation, two external momenta)
  Lattice calculations result in expansion of d-dimensional lattice integrals in external momenta and a "continuum" calculation in dim. regularisation such that the $1/\epsilon$-poles have to cancel
  Some semi-analytic approaches known for the continuum part (loop diagrams with three different propagators): Davydychev, Tarasov, Campbell, ...

- Complete computation of diagrams in symbolic terms
  - Free Lorentz index structure $\rightarrow$ construct all possible representations

- Number of analytic and numeric checks:
  Analytic cancellation of pole terms
  Recover results for forward case
- Decoupling between symbolic computation of diagrams and numeric computations of lattice integrals
- Expensive in CPU time and memory

Operator Feynman rules

Two realization of operators with derivatives $\overleftrightarrow{D}$ at momentum transfer q

(I) q at lattice position x

$$\left(\bar{\psi}\overleftrightarrow{D}_\mu\psi\right)(q) = \sum_x \left(\bar{\psi}\overleftrightarrow{D}_\mu\psi\right)(x)\ \mathrm{e}^{iq\cdot x}$$
$$= \frac{1}{2a}\sum_x \left[\bar{\psi}(x)U_{x,\mu}\psi(x+a\hat{\mu}) + \bar{\psi}(x+a\hat{\mu})U^\dagger_{x,\mu}\psi(x)\right]\left[\mathrm{e}^{iq\cdot x} + \mathrm{e}^{iq\cdot(x+a\hat{\mu})}\right]$$

(II) q at "center of mass" lattice position (Rakow)

$$\left(\bar{\psi}\overleftrightarrow{D}_\mu\psi\right)(q)$$
$$= \frac{1}{2a}\sum_x \left[\bar{\psi}(x)U_{x,\mu}\psi(x+a\hat{\mu}) + \bar{\psi}(x+a\hat{\mu})U^\dagger_{x,\mu}\psi(x)\right]\mathrm{e}^{iq\cdot(x+a\hat{\mu}/2)}$$

similar for operators with total derivatives ∂

$\longrightarrow$ e.g. $O(g^0)$ $(q = p - p' \neq 0)$:

$$\begin{aligned}
\mathcal{O}^{(DD,I)}_{\mu\nu\omega} &= \cos\frac{aq_\nu}{2}\cos\frac{aq_\omega}{2}\,\mathcal{O}^{(DD,II)}_{\mu\nu\omega} \\
\mathcal{O}^{(DD,II)}_{\mu\nu\omega} &= \frac{1}{a^2}\bar{\psi}(p')\gamma_\mu\psi(p)\sin\left(\frac{a}{2}(p+p')_\nu\right)\sin\left(\frac{a}{2}(p+p')_\omega\right)
\end{aligned}$$

Operators and mixing

Well known that operators of second and higher moments mix:

One-loop result for a matrix element of a certain operator contains structures which differ from its own Born structure

$\longrightarrow$ No multiplicative renormalisation of operators

Set of possible operators determined by the transformation properties under H(4) and charge conjugation

Only operators can mix which belong to the same representation and with identical charge conjugation number (Göckeler et al., 1996)

Mixing sets

Define symmetrisations:

$$\begin{aligned}
\mathcal{O}_{\{\nu_1\nu_2\nu_3\}} &= \frac{1}{6}\big(\mathcal{O}_{\nu_1\nu_2\nu_3} + \mathcal{O}_{\nu_1\nu_3\nu_2} \\
&\quad + \mathcal{O}_{\nu_2\nu_1\nu_3} + \mathcal{O}_{\nu_2\nu_3\nu_1} + \mathcal{O}_{\nu_3\nu_1\nu_2} + \mathcal{O}_{\nu_3\nu_2\nu_1}\big) \\
\mathcal{O}_{\|\nu_1\nu_2\nu_3\|} &= \mathcal{O}_{\nu_1\nu_2\nu_3} - \mathcal{O}_{\nu_1\nu_3\nu_2} \\
&\quad + \mathcal{O}_{\nu_3\nu_1\nu_2} - \mathcal{O}_{\nu_3\nu_2\nu_1} - 2\,\mathcal{O}_{\nu_2\nu_3\nu_1} + 2\,\mathcal{O}_{\nu_2\nu_1\nu_3} \\
\mathcal{O}_{\langle\langle\nu_1\nu_2\nu_3\rangle\rangle} &= \mathcal{O}_{\nu_1\nu_2\nu_3} + \mathcal{O}_{\nu_1\nu_3\nu_2} - \mathcal{O}_{\nu_3\nu_1\nu_2} - \mathcal{O}_{\nu_3\nu_2\nu_1}
\end{aligned}$$

Consider following representations (for spin-averaged case):

$\underline{\tau_2^{(4)},\ C=-1}$

$$\mathcal{O}^{DD}_{\{124\}} \quad \mathcal{O}^{\partial\partial}_{\{124\}}$$

$\underline{\tau_1^{(8)},\ C=-1}$

$$\begin{aligned}
\mathcal{O}_1 &= \mathcal{O}^{DD}_{\{114\}} - \frac{1}{2}\left(\mathcal{O}^{DD}_{\{224\}} + \mathcal{O}^{DD}_{\{334\}}\right) \\
\mathcal{O}_2 &= \mathcal{O}^{\partial\partial}_{\{114\}} - \frac{1}{2}\left(\mathcal{O}^{\partial\partial}_{\{224\}} + \mathcal{O}^{\partial\partial}_{\{334\}}\right) \\
\mathcal{O}_3 &= \mathcal{O}^{DD}_{\langle\langle 114\rangle\rangle} - \frac{1}{2}\left(\mathcal{O}^{DD}_{\langle\langle 224\rangle\rangle} + \mathcal{O}^{DD}_{\langle\langle 334\rangle\rangle}\right) \\
\mathcal{O}_4 &= \mathcal{O}^{\partial\partial}_{\langle\langle 114\rangle\rangle} - \frac{1}{2}\left(\mathcal{O}^{\partial\partial}_{\langle\langle 224\rangle\rangle} + \mathcal{O}^{\partial\partial}_{\langle\langle 334\rangle\rangle}\right) \\
\mathcal{O}_5 &= \mathcal{O}^{\partial D,5}_{||213||} \\
\mathcal{O}_6 &= \mathcal{O}^{\partial D,5}_{\langle\langle 213\rangle\rangle}
\end{aligned}$$

An additional operator is zero in one-loop

Matrix of renormalisation factors

Let $\Gamma_j^D(p', p, \mu, g_R, \epsilon)$ the dimensionally regularised vertex function of operator $\mathcal{O}_j$, $j = 1, \ldots, N$

$\bar{g}_R^2 = g_R^2 C_F/(16\pi^2)$

Renormalised vertex in one-loop in $\overline{MS}$ scheme

$$\Gamma_j^{\overline{MS}}(p', p, \mu, g_R) = \Gamma_j^{\text{Born}}(p', p) + \bar{g}_R^2 \left[\sum_{k=1}^{N} \gamma_{jk}^V \left(-\ln \frac{(p'+p)^2}{4\mu^2} \right) \Gamma_k^{\text{Born}}(p', p) + f_j(p', p) \right] + O(g_R^4)$$

Regularised vertex on the lattice (without possible $1/a^k$)

$$\Gamma_j^L(p', p, a, g_R) = \Gamma_j^{\text{Born}}(p', p) + \bar{g}_R^2 \left[\sum_{k=1}^{N} \gamma_{jk}^V \left(-\ln \frac{a^2(p'+p)^2}{4} \right) \Gamma_k^{\text{Born}}(p', p) + f_j^L(p', p) \right] + O(g_R^4)$$

A matrix ζ should relate the lattice Γ^L to $\overline{MS}$ vertex functions $\Gamma^{\overline{MS}}$

$$\Gamma_j^R = \sum_{k=1}^{N} \left(\delta_{jk} + \bar{g}_R^2 \zeta_{jk} + O(g_R^4) \right) \Gamma_k^L$$

It can be shown that: $\zeta_{jk} = \gamma^V_{jk} \ln(a^2\mu^2) - c^V_{jk}$

The p and p' independent constants c_{jk} have to fulfil

$$f^L_j(p,p') - f_j(p,p') \;=\; \sum_{k=1}^{N} c^V_{jk} \Gamma^{\mathrm{Born}}_k(p,p')$$

If the last equation cannot be satisfied, mixing operators have been overlooked

General form of connection between Γ^L and $\Gamma^{\overline{MS}}$

$$\Gamma^R_j(p,p',\mu,g_R) \;=\; Z_\psi \sum_{k=1}^{N} Z_{jk} \Gamma^L_k(p,p',a,g_R).$$

Z_ψ relating lattice to $\overline{MS}$ is known: $Z_\psi = 1 + \bar{g}^2_R \left(\ln(a^2\mu^2) + 1 - b_\psi\right)$

As result we obtain for the renormalisation mixing matrix

$$Z_{jk} = \delta_{jk} + \bar{g}^2_R \left[\left(\gamma^V_{jk} - \delta_{jk}\right) \ln(a^2\mu^2) - c^V_{jk} - \delta_{jk}(1 - b_\psi) \right] + O(g^4_R)$$

Examples for renormalisation factors

$$Z^m_{jk} \;=\; \delta_{jk} - \bar{g}_R^2 \left(\gamma_{jk} \ln(a^2\mu^2) + c^{(m)}_{jk}\right)$$

with $m = I, II$

1. $\mathcal{O}^{DD}_{\{124\}} \leftrightarrow \mathcal{O}^{\partial\partial}_{\{124\}}$

$$\gamma_{jk} \;=\; \begin{pmatrix} \frac{25}{6} & -\frac{5}{6} \\ 0 & 0 \end{pmatrix}$$

$$c^{(I,II)}_{jk} \;=\; \begin{pmatrix} -11.563 & 0.024 \\ 0 & 20.618 \end{pmatrix}$$

Numbers in red agree with forward matrix element case

2. $\{\mathcal{O}_1, ..., \mathcal{O}_6\}$, same dimension (preliminary)

$$\gamma_{jk} = \begin{pmatrix} \frac{25}{6} & -\frac{5}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{7}{6} & -\frac{5}{6} & 1 & -\frac{3}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & -\frac{2}{3} & \frac{2}{3} \end{pmatrix}$$

$$c_{jk}^{(I,II)} = \begin{pmatrix} -12.127 & -2.737/1.491 & 0.368 & 0.993/-0.416 & 0.0156 & 0.150 \\ 0 & 20.618 & 0 & 0 & 0 & 0 \\ 3.306 & 18.184/-8.015 & -14.852 & -4.302/4.302 & -0.928 & 0.738 \\ 0 & 0 & 0 & 20.618 & 0 & 0 \\ 0 & 3.264 & 0 & 0 & 0.350 & 0.0149 \\ 0 & 3.264 & 0 & 0 & 0.0050 & 0.360 \end{pmatrix}$$

3. $\mathcal{O}_1$ - $\frac{1}{a}$ part

In one-loop $\frac{1}{a}$ contributions to the matrix element of operator $\mathcal{O}_{\mu\nu\omega}$

Group theory and charge conjugation:

construct a possible candidate from the lower dimensional operator

$$\overline{\mathcal{O}}_{\mu\nu\omega} = \partial_\nu \left(\overline{\psi} \left[\gamma_\mu, \gamma_\omega \right] \psi \right)$$

Operator in the same representation as $\mathcal{O}_1$:

$$\mathcal{O}_7 = \overline{\mathcal{O}}_{114} - \frac{1}{2} \left(\overline{\mathcal{O}}_{224} + \overline{\mathcal{O}}_{334} \right)$$

Get multiplicative mixing

$$\mathcal{O}_1 \Big|_{1/a-\text{part}} = \bar{g}_R^2 (-0.518) \frac{1}{a} \mathcal{O}_7^{\text{Born}}$$

Subtract nonperturbatively from matrix element of $\mathcal{O}_1$

difficult task in simulations

Summary & outlook

- First results for perturbative Z-factors for 2nd moments of GPDs - Wilson fermion case (spin-averaged and spin-dependent)

- Mixing more complicated than for forward matrix elements

- Small mixing for the $\tau_2^{(4)}$ case (three different indices)
  Sizeable mixing – 10-20 % – for the $\tau_2^{(8)}$ case (2 equal indices)
  Additional mixing with lower dimensional operator

- Results concerning mixing are valid in general
  Limited applicability for numerical results using Wilson fermions

- Problems for the future:
  $\mathcal{O}_{\mu\nu\omega\sigma} = \overline{\psi}\,\left[\gamma_\mu, \gamma_\omega\right] \overset{\leftrightarrow}{D}_\omega \overset{\leftrightarrow}{D}_\sigma \psi$ for Wilson fermions
  Perturbative renormalisation for 2nd moment of GPDs with clover and overlap fermions